# Minimising The Waiting Time at Bank Atm For Service with Queuing Model

Statistics

**KEYWORDS :** Bank ATM, Little's theorem, M/M/1 queuing model, queue, waiting lines

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## ABSTRACT

*Bank ATMs would avoid losing their customers due to a long wait on the line. The bank initially provides one ATM in every branch. But one ATM would not serve a purpose when customers with draw to use ATM and try to use other bank ATM. Thus, to maintain the customers, the service time needs to be improved. This paper shows that the queuing theory used to solve this problem. We obtained the data from a bank ATM in a city. We then derive the arrival rate, service rate, utilization rate, waiting time in the queue and the average number of customers in the queue based on the data using Little's theorem and M/M/1 queuing model. The arrival rate at a bank ATM on Saturday during banking time is 1 customer per minute (cpm) while the service rate is 1.625 cpm. The average number of customers in the ATM is 1.6 and the utilization period is 0.617. We discuss the benefits of applying queuing theory to a busy ATM in conclusion.*

## INTRODUCTION

Queuing theory was initially proposed by A.K. Erlang in 1903. Queuing theory is the study of queue or waiting lines.

This paper uses queuing theory to study the waiting lines in Bank ATM in a city. The bank provides one ATM in every branch. In ATM, bank customers arrive randomly and the service time i.e., the time customer takes to do transaction in ATM, is also random. We use Little's theorem and M/M/1 queuing model to derive the arrival rate, service rate, utilization rate, waiting time in the queue. On average, 550 customers are served on weekends (Saturday and Sunday) and 300 customers are served on weekdays (Monday to Friday) monthly. Generally, on Saturdays, there are more customers coming to ATM during 10 am to 8 pm.

## QUEUING THEORY

In 1908, Copenhagen Telephone Company requested Agner K. Erlang to work on the holding times in a telephone switch. He identified that the number of telephone conversations and telephone holding time fit into Poisson distribution and exponentially distributed. This was the beginning of the study of queuing theory. In this section, we will discuss two common concepts in queuing theory.

### Little's Theorem

Little's theorem describes the relationship between throughput rate (i.e., arrival and service rate), cycle time and work in process (i.e., number of customers/jobs in the system). This relationship has been shown to be valid for a wide class of queuing models. The theorem states that the expected number of customers (N) for a system in steady state can be determined using the following equation.

$$L = \lambda T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Here $\lambda$, is the average customers arrival rate and T is the average service rate for a customer.

## ATM MODEL (M/M/1 QUEUING MODEL)

M/M/1 queuing model means that the arrival and service time are exponentially distributed (Poisson process). For the analysis of the ATM M/M/1 queuing model, the following variables will be investigated:

$\lambda$: The mean customers arrival rate

$\mu$: The mean service rate

$\rho = \frac{\lambda}{\mu}$ : utilization factor

Probability of zero customers in the ATM:

$$\rho_0 = 1 - \rho \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$P_n$: The probability of having n customers in the ATM:

$$P_n = P_0\rho^n = (1-\rho)\rho^n \ldots\ldots\ldots\ldots\ldots (3)$$

L: The average number of customers in the ATM:

$$L = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu - \lambda} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$L_q$: The average number of customers in the queue:

$$L_q = L\,X\,\rho = \frac{\rho^2}{1-\rho} = \frac{\rho\lambda}{\mu-\lambda} \ldots\ldots\ldots\ldots\ldots \quad (5)$$

$W_q$: The average waiting time in the queue:

$$W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu - \lambda} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

W: The average time spent in the ATM, including the waiting time:

$$W = \frac{L}{\lambda} = \frac{1}{\mu - \lambda} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

## OBSERVATION & DISCUSSION

We have collected the one month daily customer data by observation during banking time, as shown in Table-1. The data is graphically represented in Fig. 1 and 2.

**Table – 1 [Monthly Customer Counts]**

| | Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|---|---|---|---|---|---|---|---|
| 1st Week | 181 | 161 | 148 | 130 | 117 | 75 | 200 |
| 2nd Week | 161 | 134 | 127 | 119 | 95 | 73 | 170 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3rd Week | 135 | 110 | 108 | 98 | 74 | 64 | 145 |
| 4th Week | 89 | 55 | 47 | 39 | 20 | 40 | 102 |
| Total | 560 | 460 | 430 | 386 | 306 | 252 | 617 |

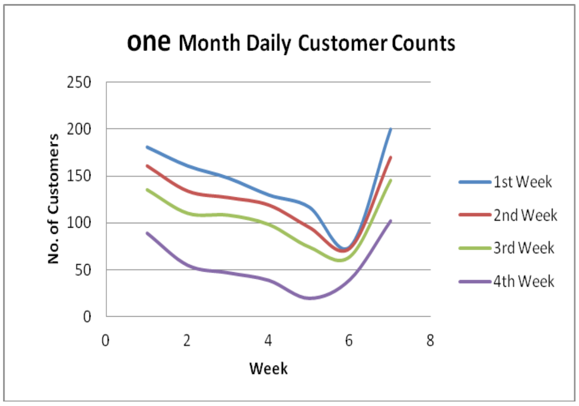


**Figure – 1**

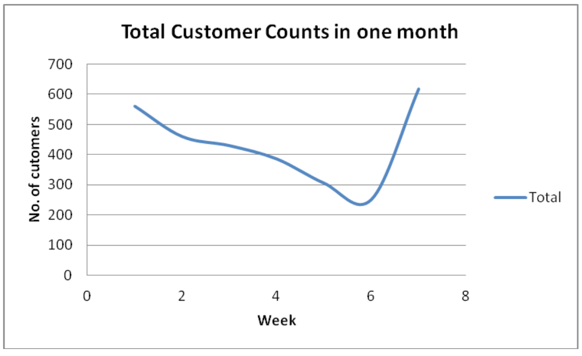


**Figure -2**

**Calculation**
We investigate that, after Friday, during first two days of a week, there are on average 60 people coming to the ATM in one hour time period of banking time. From this we can derive the arrival rate as

$\lambda = \frac{60}{60} = 1$ customer/minute (cpm)

We also found out from observation that each customer spends 1.6 minutes on average in the ATM (W), the queue length is around 3 people ($L_q$) on average and the average waiting time is around 2 minutes i.e., 120 seconds.

Theoretically, the average waiting time is

$$W_q = \frac{L_q}{\lambda} = \frac{3\ customers}{1\ cpm} = 3 \text{ minutes} = 180 \text{ seconds}$$

From this calculation, we can see that, the observed actual waiting time does not differ by much when it is compared with the theoretical waiting time.

Next, we will calculate the average number of people in the ATM using (1)

L = 1cpm X 1.6 minutes = 1.6 customers

Using (4) we can also derive the service rate and the utilization rate.

$$\mu = \frac{\lambda(1+L)}{L} = \frac{1(1+1.6)}{1.6} = 1.625cpm$$

Hence, $\rho = \frac{\lambda}{\mu} = \frac{1cpm}{1.625cpm} = 0.617$

This is the probability that, the server, in this case ATM, is busy to serve the customers, during banking time. So, during banking time, the probability of zero customers in the ATM is

$$P_0 = 1 - \rho = 1 - 0.617 = 0.383$$

The queuing theory provides the formula to calculate the probability of having n customer in the ATM as follows:

$$P_n = (1-\rho)\rho^n = (1-0.617)(0.617)^n = (0.383)(0.617)^n$$

We assume that impatient customers will start to balk when they see more than 3 people are already queuing for the ATM. We also assume that the maximum queue length that a patient customer can tolerate is 10 people. As the capacity of the ATM is 1 people, we can calculate the probability of 4 people in the system (i.e., in the ATM)

Therefore, the probability of customers going away =

P (more than 3 people in the queue) = P (more than 4 people in the ATM) is

$$P_{5-11} = \sum_{n=5}^{11} P_n = 0.08632 = 8.63\%$$

**Evaluation**

- The utilization is directly proportional with the mean number of customers. It means that the mean number of customers will increase as the utilization increase.
- The utilization rate at the ATM is very high at 0.617. This, however, is only the utilization rate during banking time on Saturday and Sunday. On weekdays, the utilization rate is almost half of it. This is because the number of customers on weekdays is only half of the number of people on weekends.
- In case of the customers waiting time is lower or in other words, we waited for less than 120 seconds, the number of customers that are able to be served per minute will increase. When the service rate is higher, the utilization will be lower, which makes the probability of the customers going away decreases.

**Benefits**
This research can help bank ATM to increase its QoS (Quality of Service), by anticipating, if there are many customers in the queue. The result of this paper work may become the reference to analyze the current system and improve the next system. Because the bank can now estimate the number of customers waits in the queue and the number of customers going away each day.

By estimating the number of customers coming and going in a day, the bank can set a target that, how many ATMs are required to serve people in the main branch or any other branch of the bank.

**CONCLUSION**
This research paper has discussed the application of queuing theory to the Bank ATM. From the result we have obtained that the rate at which customers arrive in the queuing system is 1 customer per minute and the service rate is 1.625 customers per minute. The probability of buffer flow if there are 3 or more customers in the queue is 8 out of 100 customers. The probability of buffer overflow is the probability that customers will run away, because may be they are impatient to wait in the queue. This theory is also applicable for the bank, if they want to calculate all the data daily and this can be applied to all branches ATM also.

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